MAC-CPTM Situations Project

Harrington Situation 1: Absolute Value Defined Piecewise

Prompt

In a middle school Advanced Algebra class, students have been learning about function notation, domain, range, modeling functions with graphs, tables and rules when the teacher writes f(x) = | x | on the board and asks if students could think of a way to write a rule that would apply for x ≥ 0 and another for x-values less than 0.

$$f(x) = \begin{cases} \_\_\_\_\_\_\_\_\_\_, & \text{for } x \geq 0 \\ \_\_\_\_\_\_\_\_\_\_, & \text{for } x < 0 \end{cases}$$

The class quickly decided that if x ≥ 0, the rule would just be f(x) = x, but were stumped for x < 0. The teacher asked, "What if x = -3?" After some thought, a student suggest that you do -2 times the number plus the number and the teacher writes -2(-3) + -3. The teacher asks if this will always work and the student says, "yes, -2(-5) + -5 = 5."

Mathematical Foci

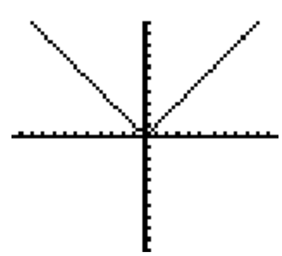
Mathematical Focus 1

Absolute value is a distance from 0.

The absolute value of x is the distance x is from 0.

Mathematical Focus 2

The function f(x) = |x| is two linear functions, f(x) = x for x≥0, and f(x)=-x for x<0.



Mathematical Focus 3

The breakpoint for an absolute value function is the value that makes the expression inside the absolute value 0.

For $f(x) = | g(x) |$, $f(x) = g(x)$ when $g(x) \geq 0$ and $f(x) = -g(x)$ when $g(x) < 0$.

If $f(x) = | x - 2 |$, then $f(x) = x - 2$ when $x - 2 \geq 0$ or $x \geq 2$ and $f(x) = -(x - 2)$ when $x < 2$.

Mathematical Focus 4

Proof is not accomplished by example and middle school Algebra students can begin to develop an understanding of how they might develop a mathematical argument to show that something is always true.

In this specific case, the pattern observed is: $-2(-3) + -3 = 3$
$-2(-5) + -5 = 5$
$-2(-8) + -8 = 8$
so the very simple conjecture is “proved” $-2(x) + x = -x$